MODELLI ATTUARIALI DETERMINISTICI PER LE ASSICURAZIONI VITA

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Filo conduttore: Equazioni di bilancio

- Descrivono l'equilibrio attuariale, su base annuale, di una gestione assicurativa vita
- Modello sottostante deterministico (presupposto: piena realizzazione dell'effetto pooling)
- Il paradigma su cui sono basate è una guida anche in contesti più sfaccettati o complessi
- In particolare, a fini di:
  - Progettazione di benefici a fronte di "nuovi" rischi
  - Definizione di indici per la misurazione di varie componenti di un contratto

Contents

1 Background

2 Balance equations and classical implementations
- Balance equations
- Implementation: Assessment of the size and “direction” of mutuality
- Implementation: Risk and saving
- Implementation: Profit assessment

3 Implementation: Policy design
- Linking annuity benefits to (financial and) mortality/longevity indexes

4 Implementation: User-friendly performance metrics
- For Insurance-based investment products
- For Mortality/Longevity-linked annuities

Groundwork I

Reserving

- At any point in time, for each contract there must be an actuarial balance between future premiums and benefits
- ...that is not automatically fulfilled, for instance because of the financing condition
- The reserve measures the insurer's debt and ensures such an actuarial balance (reserve $+$ actuarial value of future premiums $=$ actuarial value of future benefits)

Actuarial value (and actuarial balance) defined consistently with the reference regulation

Groundwork II

(Net premium) Policy reserve

Considering a policy in-force at time t (policy anniversary), following local GAAP, it is defined as follows:

$$V_t = \underbrace{\mathrm{Ben}(t, m)}_{\substack{\text{Actuarial value at} \\ \text{time } t \text{ of the bene-} \\ \text{fits in } (t, m)}} - \underbrace{\mathrm{Prem}(t, m)}_{\substack{\text{Actuarial value at} \\ \text{time } t \text{ of the pre-} \\ \text{miums in } (t, m)}}$$

- It ensures the actuarial balance in (t, m)

  $$\Rightarrow \underbrace{V_t + \mathrm{Prem}(t, m)}_{\text{Resources in } (t, m)} = \underbrace{\mathrm{Ben}(t, m)}_{\text{Obligations in } (t, m)}$$

- Risk margin embedded in the actuarial value, via choice of the (reserving) parameters

Notation

Refer to a (general) insurance contract with:

- Maturity: m
- Entry age: x
- Death benefit at time t: C_t
- Maturity (survival) benefit: S
- Annual premium at time t: P_t

Specific policy structures

Follows from an appropriate choice of the benefit or premium amount

$\Rightarrow$ Apart from life annuities in arrears, all possible arrangements with life-contingent benefits are represented

Balance equations I

Policy reserve at time t

$$
\begin{aligned}
V_t &= \mathrm{Ben}(t,m) - \mathrm{Prem}(t,m) \\
&= \sum_{h=0}^{m-t-1} C_{t+h+1} \cdot (1+i)^{-(h+1)} \cdot {}_{h|1}q_{x+t} + S \cdot (1+i)^{-(m-t)} \cdot {}_{m-t}p_{x+t} - \sum_{h=0}^{m-t-1} P_{t+h} \cdot (1+i)^{-h} \cdot {}_{h}p_{x+t}
\end{aligned}
$$

Thanks to some little (actuarial) algebra, we obtain the recursion

$$
V_t + P_t = C_{t+1} \cdot (1+i)^{-1} \cdot q_{x+t} + V_{t+1} \cdot (1+i)^{-1} \cdot p_{x+t} \tag{1}
$$

Balance equations II

Alternative expressions

$$V_t + P_t = C_{t+1} \cdot (1+i)^{-1} \cdot q_{x+t} + V_{t+1} \cdot (1+i)^{-1} \cdot p_{x+t} \tag{1}$$

$$(V_t + P_t) \cdot (1+i) = C_{t+1} \cdot q_{x+t} + V_{t+1} \cdot p_{x+t} \tag{2}$$

$$V_t + P_t = (C_{t+1} - V_{t+1}) \cdot (1+i)^{-1} \cdot q_{x+t} + V_{t+1} \cdot (1+i)^{-1} \tag{3}$$

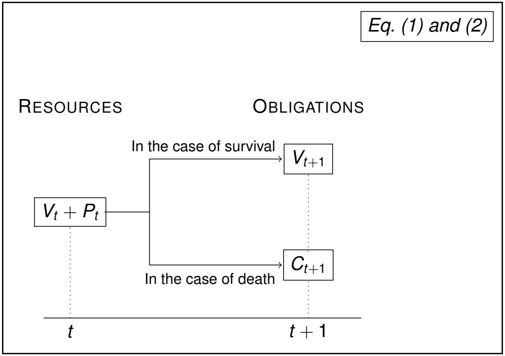
$$(V_t + P_t) \cdot (1+i) = \underbrace{(C_{t+1} - V_{t+1})}_{\text{sum at risk}} \cdot q_{x+t} + V_{t+1} \tag{4}$$

Eq. (1): Fouret equation (1891)

Eq. (4): Kanner equation (1869)

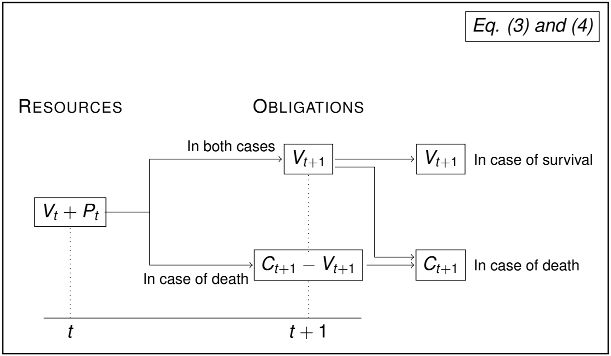
Balance equations III

Interpretation



Balance equations IV

Interpretation



Assessment of the size and “direction” of mutuality

From balance equation (4)

$$\underbrace{V_{t+1} - V_t}_{\text{Variation of the reserve}} = \underbrace{P_t}_{\text{Premium}} + \underbrace{(V_t + P_t) \cdot i}_{\text{Interest}} - \underbrace{(C_{t+1} - V_{t+1}) \cdot q_{x+t}}_{\text{Mutuality}}$$

- Policy design (for instance, IBIPs)
- A key to describe the policy fees (see later)

Risk and saving premium I

From balance equation (3)

$$P_t = \underbrace{(C_{t+1} - V_{t+1}) \cdot (1+i)^{-1} \cdot q_{x+t}}_{\text{Risk premium, } P_t^{[R]}} + \underbrace{V_{t+1} \cdot (1+i)^{-1} - V_t}_{\text{Saving premium, } P_t^{[S]}}$$

Risk and saving premium II

Saving premiums

Maintain the reserving process

$$V_{t+1} = (V_t + P_t^{[S]}) \cdot (1+i) = \sum_{h=0}^{t} P_h^{[S]} \cdot (1+i)^{t+1-h}$$

Reserve $=$ Result of the financial accumulation of the saving premiums

Risk premium

$=$ Premium of a one-year term insurance with benefit amount the sum at risk (Natural premium for the sum at risk)

Risk and saving for a life annuity I

Single-premium life annuity in arrears

- Balance equation

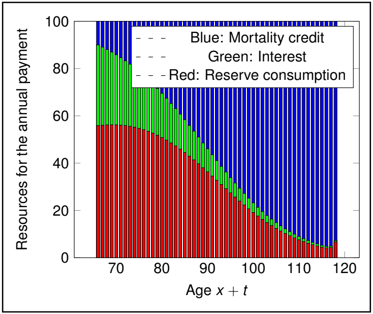
$$(V_t + P_t) \cdot (1 + i) = (V_{t+1} + b) \cdot p_{x+t}$$

Where: $P_0 = \Pi = b \cdot a_x$, $P_1 = P_2 = \cdots = 0$

- A splitting of the benefit amount

$$b = \underbrace{V_t - V_{t+1}}_{\text{Reserve "consumption"}} + \underbrace{V_t \cdot i}_{\text{Interest}} + \underbrace{(V_{t+1} + b) \cdot q_{x+t}}_{\text{Mortality credit (Mutuality)}}$$

Risk and saving for a life annuity II



- Insights into the timing of financial and longevity risk

Expected annual profits I

From balance equation (2)

$$\underbrace{(V_t + P_t) \cdot (1 + i) - C_{t+1} \cdot q_{x+t} - V_{t+1} \cdot p_{x+t}}_{\text{TB1}} = 0$$

- The actuarial balance relies on the adoption of the same (conservative) technical basis, namely TB1, in all the elements

In realistic terms

- Realistic return on investments: i''
- Realistic mortality rate: q''_x

$\boxed{\Rightarrow}$ Scenario basis TB2 for some elements

Expected annual profits II

Realistic comparison between resources and obligations

TB2

$$(V_t + P_t) \cdot (1 + i'') - C_{t+1} \cdot q''_{x+t} - V_{t+1} \cdot p''_{x+t} = \overline{PL}_{t+1}$$

TB1

$\overline{PL}_{t+1}$ ($\gtreqless 0$): Expected annual profit/loss, per policy in-force (*under a deferral & matching approach*)

Clearly, a different algebraic expression for the annual expected profit when the risk margin in the reserve is computed explicitly

A case of current interest: Post-retirement benefits

The framework

- Individual wealth is increasingly exposed to *financial and inflationary risk* (over the whole lifetime) and *(individual) longevity risk* (after retirement)

- *Individual longevity risk*

  $=$ *Risk of outliving retirement wealth*

- The standard annuity (SPIA – Single Premium Immediate Annuity) provides protection, but it is still an unpopular product

Standard annuity

Lifelong payment (fixed or minimum annual amount)

- Independent of: Individual's lifetime *&* Average population lifetime *&* Returns on investment *&* Inflation rates
- Relying on guaranteed mortality credits (and a guaranteed – minimum – return)

Provider

- Exposure to several risks (financial, idiosyncratic & aggregate longevity) over a long-term time-horizon
- Adverse selection
- Pricing assumptions, and all features of the annuity design, chosen at issue, without following updates
  - Conservative assumptions $\Rightarrow$ *Loadings*
  - *Inflexible benefits* (apart from participation to extra-returns)

Individual

- Lifelong protection (but not to inflation)
- Mortality credits $\Rightarrow$ No bequest
- Irreversible decision
- Illiquid asset
  - Pre-set benefits
  - No (partial) surrenders
  - Assets chosen by provider
- In general: Short-sighted gaze
  - Greater attention to early death (both at micro and macro level)
  - However, longevity risk is long-term

Among the possible innovations in the annuity design: Relaxing the guarantees

Postpone the start of the benefit payment

- Old age annuities (possibly deferred)

Limit the number of payments

- Temporary annuities (Extendable annuities)
- Guaranteed Minimum Withdrawal benefits

Link the benefit amount to a given mortality/longevity experience

☞ *Mortality/longevity-linked annuity benefits*

- Guarantees are not (necessarily) excluded (for example: a minimum benefit amount), but should be cheaper

Mortality/Longevity-linking

Participating structure

- The benefit amount is allowed to fluctuate (up or down), depending on a chosen longevity experience
- Guarantees can be included (for example: a minimum benefit amount)

Benefit at time t

$$b_t = b_{t-1} \cdot \mathrm{adj}_t$$

- adj_t: Adjustment coefficient at time t (possibly applied every k years), expressing a longevity experience over a convenient time-interval

To define the adjustment coefficient

We need

1. A mortality/longevity experience/index
2. Quantities recording the longevity experience

Alternatives

	Portfolio/Indemnity-based	*Index-based*
Number of survivors or Survival rates (observed vs expected)	In the pool	In a reference population
Actuarial quantities	Required portfolio reserve vs Available assets	Actuarial value of the annuity with updated life tables

Indemnity vs index-based solutions

Portfolio/pool experience

- Indemnity-based solution
- No basis risk for the provider
- Subject to random fluctuations
- Vulnerable to manipulations (or perceived as such)
- Natural choice in self-insured arrangements

Experience of a reference population

- Index-based solution
- Basis risk for the provider
- Less subject to random fluctuations
- Perhaps more trusted, as it is reported by an independent institution
- Appropriate choice in insurance-based arrangements

(Projected) Life table

- Index-based solution
- Less subject to random fluctuations

Self-insured vs Insurance-based arrangements

Self-insured

- Group Self-Annuitization (GSA), Pooled/Tontine/Survivors funds/annuities
- They rely on mortality credits, but their amount is not guaranteed
- No financial guarantees

Insurance-based arrangements

- Annuities, with a benefit linked to a mortality/longevity index
- Mortality credits are partially guaranteed
- They should also include financial guarantees

Structure of the adjustment coefficient: A general framework I

What follows refers to:

- A life annuity immediate
- One cohort
- Entry time: 0. Entry age: x

Initial benefit amount

$$b_0 = S \cdot \frac{1}{a_x(0) \cdot (1 + \pi)}$$

- $a_x(0)$: Actuarial value at time 0 of a unitary annuity, based on the best-estimate assumptions at time 0
- π: Premium loading (whose size should be justified by that of the longevity guarantee)
- S: Initial capital (paid by the individual)

Structure of the adjustment coefficient: A general framework II

Recursion for the reserve in year $(t-1, t)$ ...

(One policy, in-force at time $t-1$)

... According to the reserving basis adopted at time $t-1$

$$\overbrace{b_{t-1} \cdot a_{x+t-1}(\tau)}^{\text{Reserve at time } t-1} \cdot (1 + i(\tau)) = \underbrace{b_{t-1} \cdot (1 + a_{x+t}(\tau)) \cdot p_{x+t-1}(\tau)}_{\text{Payment + Reserve at time } t \text{, if alive}}$$

where

- τ: Time at which the reserving basis has been chosen, $0 \leq \tau \leq t-1$
- $a_{x+t-1}(\tau), a_{x+t}(\tau)$: Actuarial value at time $t-1$ and t of a unitary annuity, based on the best-estimate assumptions at time τ
- $p_{x+t-1}(\tau)$: Survival rate from the life table chosen at time τ
- $i(\tau)$: Technical interest rate chosen at time τ

Structure of the adjustment coefficient: A general framework III

However

- If there is a financial linking: the return assigned to the reserve in year $(t-1, t)$ is g_t (hopefully, higher than $i(\tau)$)
- If there is a longevity linking:
  - The survival rate could be measured in a chosen population (either the portfolio or a reference population) $\Rightarrow \tilde{p}_{x+t-1}$ instead of $p_{x+t-1}(\tau)$
  - The reserving basis in the actuarial value of the annuity at time t could be updated to a later time $\tau' \Rightarrow a_{x+t}(\tau')$ instead of $a_{x+t}(\tau)$, $\tau \leq \tau' \leq t$

Then: The actuarial balance is kept by adjusting the benefit amount to b_t

Structure of the adjustment coefficient: A general framework IV

Actuarial balance in year $(t-1, t)$ ...   (One policy, in-force at time $t-1$)

... in terms of the conditions applied to the annuitant

$$\underbrace{\overbrace{b_{t-1} \cdot a_{x+t-1}(\tau)}^{\text{Reserve invested at time } t-1} \cdot (1+g_t)}_{\text{"Assets" at time } t} = \underbrace{b_t \cdot (1 + a_{x+t}(\tau')) \cdot \tilde{p}_{x+t-1}}_{\text{Payment + Reserve at time } t\text{, if alive}}$$

☞ $b_t \gtreqless b_{t-1}$

Structure of the adjustment coefficient: A general framework V

Benefit at time t

$$b_t = b_{t-1} \cdot \frac{\overbrace{a_{x+t-1}(\tau) \cdot (1+g_t)}^{\text{Available assets}}}{\underbrace{(1+a_{x+t}(\tau')) \cdot \tilde{p}_{x+t-1}}_{\text{(Payment +) Required reserve}}}$$

- Typical structure in self-insured arrangements or when no guarantee is provided (e.g., GSA)
- In this case:

$\tau = t-1$	Latest best-estimate
$\tau' = t$	Current best-estimate
$\tilde{p}_{x+t-1} = \tilde{p}^{[\text{ptf}]}_{x+t-1}$	Observed in the pool $\Rightarrow$ Indemnity-based
$g_t = \tilde{\imath}_t$	Realized return

Structure of the adjustment coefficient: A general framework VI

Equivalently: Benefit at time t

$$b_t = b_{t-1} \cdot \underbrace{\frac{1+g_t}{1+i(\tau)}}_{\substack{\text{Return}\\ \text{on investments}}} \cdot \underbrace{\frac{p_{x+t-1}(\tau)}{\tilde{p}_{x+t-1}}}_{\text{Survival rate}} \cdot \underbrace{\frac{1+a_{x+t}(\tau)}{1+a_{x+t}(\tau')}}_{\substack{\text{Actuarial value}\\ \text{of the annuity}}}$$

- Appropriate structure in insurance-based arrangements
- In this case, it is also appropriate to link the mortality/longevity adjustment only to the survival rate or only to the actuarial value of the annuity $\Rightarrow$ Some risk is retained by the provider

Choices: Financial linking I

$$b_t = b_{t-1} \cdot \frac{1+g_t}{1+i(\tau)}$$

- Numerator: g_t Credited to the policy reserve

  $g_t = i(0)$ Fixed return (guaranteed)
  $g_t = \tilde{\imath}_t$ Realized return on assets (no guarantee)
  $g_t \geq i_{\min}$ Minimum return guaranteed

- Denominator $i(\tau)$: Best-estimate assumpion at time τ ($\rightsquigarrow$ Benchmark interest rate)

  $\tau = 0$ Initial best-estimate
  $\tau = t - k$ Periodic revision of the benchmark
  $\tau = t - 1$ Latest best-estimate

- For example, in the traditional participating arrangement: $\frac{1+g_t}{1+i(0)} = \max\left\{1 + r_{\min}, \frac{1+\eta\tilde{\imath}_t}{1+i(0)}\right\}$

Choices: Linking based on the survival rate I

$$b_t = b_{t-1} \cdot \frac{p_{x+t-1}(\tau)}{\tilde{p}_{x+t-1}}$$

- Denominator: $\tilde{p}_{x+t-1}$ Assigned to the policy reserve

$\tilde{p}_{x+t-1} = p_{x+t-1}(0)$	Guaranteed mortality/longevity (at time 0)
$\tilde{p}_{x+t-1} = \tilde{p}^{[\text{ptf}]}_{x+t-1}$	Observed in the portfolio $\Rightarrow$ Indemnity-based, without guarantees
$\tilde{p}_{x+t-1} = \tilde{p}^{[\text{pop}]}_{x+t-1}$	Obs. in a reference population $\Rightarrow$ Index-based, without guarantees

- Numerator: $p_{x+t-1}(\tau)$ Best-estimate assumption at time τ ($\rightsquigarrow$ Benchmark survival rate)

$\tau = 0$	Initial best-estimate
$\tau = t - k$	Periodic revision of the benchmark
$\tau = t - 1$	Latest best-estimate

Choices: Linking based on the survival rate II

Particular choices

- $b_t = b_{t-1} \cdot \frac{p_{x+t-1}(0)}{\tilde{p}^{[\text{pop}]}_{x+t-1}} = \cdots = b_0 \cdot \frac{{}_tp_x(0)}{{}_t\tilde{p}^{[\text{pop}]}_x}$ Benchmark: Best-estimate at time 0
- $b_t = b_{t-1} \cdot \frac{p_{x+t-1}(t-1)}{\tilde{p}^{[\text{pop}]}_{x+t-1}}$ Benchmark: Latest best-estimate

Choices: Linking based on the survival rate III

Guarantees

Can be introduced by setting minimum/maximum values for

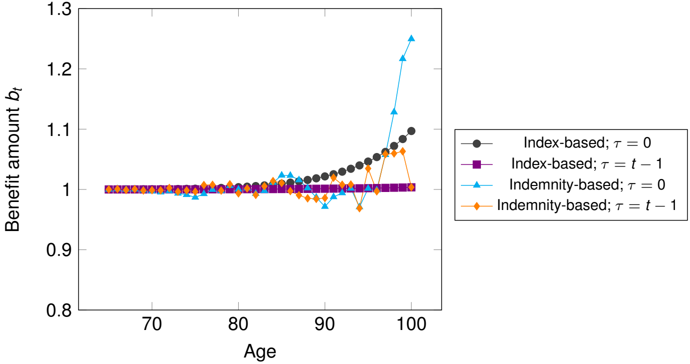
- The ratio $\frac{p_{x+t-1}(\tau)}{\tilde{p}_{x+t-1}}$
- The probabilities $\tilde{p}_{x+t-1}$
- The benefit amount b_t

  Such bounds can also serve to avoid the transfer of major profits
- Adjustments up to a given age
- Partial participation

Choices: Linking based on the survival rate IV

Example of possible trajectories of the benefit amount

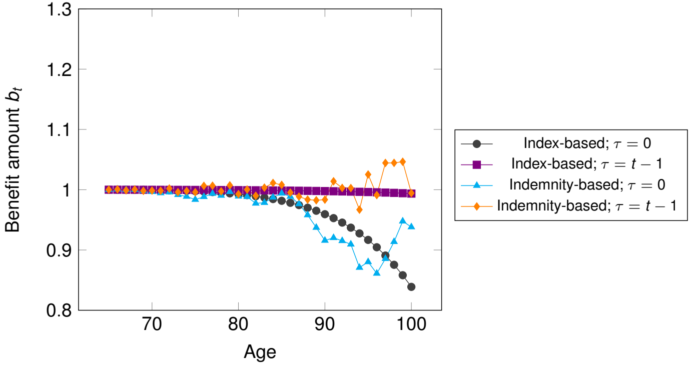
- Experienced mortality on average higher than expected, with major random fluctuations in the small population



Choices: Linking based on the survival rate V

Example of possible trajectories of the benefit amount

- Experienced mortality is on average lower than expected, with major random fluctuations in the small population



Choices: Linking based on the actuarial value of the annuity I

$$b_t = b_{t-1} \cdot \frac{1+a_{x+t}(\tau)}{1+a_{x+t}(\tau')}$$

- Particular choices:

  $\tau = 0, \tau' = t$: $\quad b_t = b_0 \cdot \frac{1+a_{x+t}(0)}{1+a_{x+t}(t)}$

  $\tau = t-1, \tau' = t$: $\quad b_t = b_{t-1} \cdot \frac{1+a_{x+t}(t-1)}{1+a_{x+t}(t)}$

- Possible guarantees:

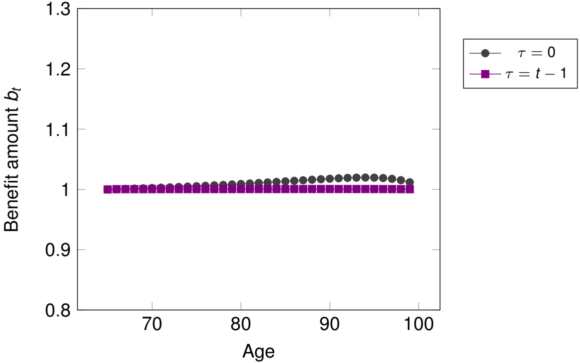
  Minimum (and maximum) value for the benefit amount, maximum age for the adjustment, partial participation

- Possible approximation: $b_t = b_{t-1} \cdot \frac{a_{x+t}(\tau)}{a_{x+t}(\tau')}$

Choices: Linking based on the actuarial value of the annuity II

Example of possible trajectories of the benefit amount

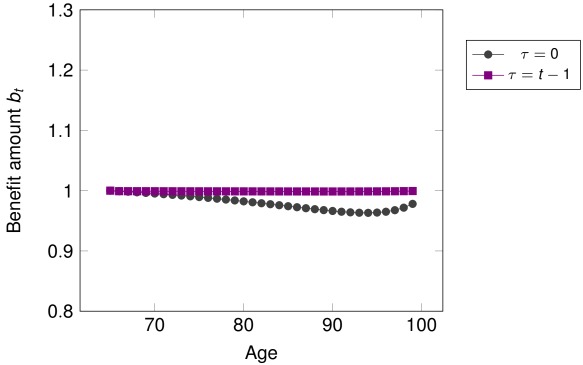
- Updated life tables predict on average a lower expected lifetime



Choices: Linking based on the actuarial value of the annuity III

Example of possible trajectories of the benefit amount

- Updated life tables predict on average a higher expected lifetime



Some results: Premium loading I

Basic parameters

- One cohort
- Initial age: $x = 65$. Maximum attainable age: $\omega = 100$
- No financial return, no financial risk (and no financial linking)
- Annuity immediate
- Stochastic mortality rate

Some results: Premium loading II

Arrangements

- Fixed benefit
- GSA arrangement
- Linking based on the survival rates (L-SP)
  - Mortality experience measured in a reference population (index-based linking)
  - Benchmark survival rate: either the best-estimate at time 0 or the latest best-estimate
  - Maximum age for benefit adjustment: $x_{\text{max}} = 95$
  - Maximum reduction of the benefit amount (in respect of the initial amount): 25%
  - No uplift in respect of the initial benefit
- Linking based on the actuarial value of the annuity (L-AV)
  - Benchmark actuarial value: either the best-estimate at time 0 or the latest best-estimate
  - Other conditions as above
- Adjustment every $k = 1, 3, 5$ years

Some results: Premium loading III

Premium loading

- Assessed such that the provider's probability of loss is 10%, excluding basis risk

  (the premium loading is then expressed as a % of the actuarial value of a unitary annuity, based on the best-estimate assumption at time 0)

Benefit type		Moderate longevity risk	Major longevity risk
FB	Fixed benefit	1.731%	5.647%
L-SP$(t-k), k=1$	Survival rate (Benchmark: BE k years before)	1.654%	5.472%
L-SP$(t-k), k=3$	Adjustment every k years	1.572%	5.158%
L-SP$(t-k), k=5$		1.481%	4.848%
L-AV$(t-k,t), k=1$	Actuarial value (Benchmark: BE k years before)	0.092%	0.219%
L-AV$(t-k,t), k=3$	Adjustment every k years	0.185%	0.539%
L-AV$(t-k,t), k=5$		0.293%	0.892%
L-SP$(0), k=1$	Survival rate (Benchmark: BE at time 0)	0.052%	0.169%
L-SP$(0), k=3$	Adjustment every k years	0.227%	0.714%
L-SP$(0), k=5$		0.384%	1.208%
L-AV$(0,t), k=1$	Actuarial value (Benchmark: BE at time 0)	-0.034%	-0.136%
L-AV$(0,t), k=1$	Adjustment every k years	0.017%	-0.027%
L-AV$(0,t), k=1$		0.144%	0.404%
GSA	Group Self-Annuitization	0.000%	0.000%

Background and motivation I

Focus: Insurance-based investment products

- Investments ✚ Life-contingent benefits

Liability-driven

(Participating policies, life annuities)

Life-contingent benefits & Investment

- *Pricing*: Actuarial equivalence over the whole policy duration (single, level, single recurrent premiums)
- *Reserving*: Prospective reserve

⇝ Pricing and reserving understood by the policyholder?

⇝ Clearly (or intuitively) explained by the insurer?

Asset-driven

(Unit-linked policies, variable annuities)

Investment & Life-contingent benefits

- *Pricing*: Initial (Single recurrent premiums) and periodic fees
- *Reserving*: Asset value

⇝ Greater disclosure

⇝ More intuitive language

Background and motivation II

The concept of actuarial fairness / equivalence can be out of reach for many individuals

- *How can we describe more intuitively the reserving process to the individual, in particular in the case of a liability-driven arrangement?*
- *How can we provide information to the individual in an intuitive way, in line with summary return / cost indexes adopted for other financial products?*

Background and motivation III

The policyholder has usually access to information about

- The policy reserve amount (because of participation and surrendering)
- The return on the asset backing the reserve (because of participation)

Additionally,

- (S)he can gain proxy information about the realized mortality from mortality indexes

➔ Based on this information, the reserving (and, then, the pricing) process can be reinterpreted in terms of (embedded, or equivalent) periodic fees

Background and motivation IV

In the following

- Only mutuality costs and safety loadings (expenses and asset management fees are disregarded)
- Reference is always to a policy in-force at the policy anniversary
- Some trajectories of equivalent fees (deterministic setting)

Equivalent periodic fees and components I

Reserving process from the point of view of the policyholder

Policy in-force at time t

$$V_t = (\underbrace{V_{t-1}}_{\text{policy reserve}} + \underbrace{\pi_{t-1}}_{\text{net premium}}) \cdot (1 + \underbrace{i_t^{[\text{net}]}}_{\text{annual return, net of the cost of mutuality and loadings}})$$

Equivalent periodic fees and components II

Cost of mutuality, based on a mortality index

$$(V_{t-1} + \pi_{t-1}) \cdot \underbrace{\xi_t^{[\text{mut}]}}_{\text{(embedded) fee for mutuality}} = \underbrace{(C_t - V_t)}_{\text{sum at risk}} \cdot \underbrace{q_{x+t-1}^{[\text{ref}]}}_{\text{mortality rate in a reference population}}$$

Note: The mortality rate realized in the insurer's portfolio, $q_{x+t-1}^{[\text{ptf}]}$, can be other than $q_{x+t-1}^{[\text{ref}]}$

Equivalent periodic fees and components III

(Embedded) fee for loadings

Let i_t be the return on assets backing the reserve (usually disclosed to the policyholder)

$$\underbrace{\xi_t^{\text{[load]}}}_{\text{(embedded) fee for loadings}} = i_t - i_t^{\text{[net]}} - \xi_t^{\text{[mut]}}$$

Note: $\xi_t^{\text{[load]}}$ can overestimate or underestimate the loading actually charged by the insurer, depending on $q_{x+t-1}^{\text{[ptf]}}$ *vs* $q_{x+t-1}^{\text{[ref]}}$

Equivalent periodic fees and components IV

Remarks

- Having access to the reserving basis allows us to split the embedded loading into a financial ($\xi_t^{\text{[load,fin]}}$) and mortality ($\xi_t^{\text{[load,mort]}}$) component
- Some implementations for a participating endowment

Some numerical insights I

Participating endowment, level premiums

Entry age: 50
Duration: 10 years
Annual premium amount: 1,000.00 euro
Pricing interest rate: $i = 2\%$
Pricing life table: period, population life table (mortality rates q)
Annual revaluation rate of the reserve: $r_t = \max\left\{\frac{0.95 \cdot i_t - i}{1+i}, 0\right\}$
Initial benefit amount: 11,018.89 euro

One trajectory for i_t and $q^{[\mathrm{ref}]}_{x+t}$

t	i_t	$\frac{q^{[\mathrm{ref}]}_{x+t-1}}{q_{x+t-1}}$	$i^{[\mathrm{net}]}_t$	$\xi^{[\mathrm{mut}]}_t$	$\xi^{[\mathrm{load}]}_t$	$\xi^{[\mathrm{load,fin}]}_t$	$\xi^{[\mathrm{load,mort}]}_t$
1	2.000%	1.0000	0.031%	1.969%	0.000%	0.000%	0.000%
2	2.000%	1.0000	1.029%	0.971%	0.000%	0.000%	0.000%
3	2.000%	1.0000	1.374%	0.626%	0.000%	0.000%	0.000%
4	2.000%	1.0000	1.559%	0.441%	0.000%	0.000%	0.000%
5	2.000%	1.0000	1.678%	0.322%	0.000%	0.000%	0.000%
6	2.000%	1.0000	1.766%	0.234%	0.000%	0.000%	0.000%
7	2.000%	1.0000	1.833%	0.167%	0.000%	0.000%	0.000%
8	2.000%	1.0000	1.893%	0.107%	0.000%	0.000%	0.000%
9	2.000%	1.0000	1.946%	0.054%	0.000%	0.000%	0.000%
10	2.000%	1.0000	2.000%	0.000%	0.000%	0.000%	0.000%

Some numerical insights II

Participating endowment, level premiums

Entry age: 50
Duration: 10 years
Annual premium amount: 1,000.00 euro
Pricing interest rate: $i = 2\%$
Pricing life table: period, population life table (mortality rates q)
Annual revaluation rate of the reserve: $r_t = \max\left\{\frac{0.95 \cdot i_t - i}{1+i}, 0\right\}$
Initial benefit amount: 11,018.89 euro

One trajectory for i_t and $q_{x+t}^{[\text{ref}]}$

t	i_t	$\frac{q_{x+t-1}^{[\text{ref}]}}{q_{x+t-1}}$	$i_t^{[\text{net}]}$	$\xi_t^{[\text{mut}]}$	$\xi_t^{[\text{load}]}$	$\xi_t^{[\text{load,fin}]}$	$\xi_t^{[\text{load,mort}]}$
1	2.256%	0.9196	0.171%	1.811%	0.274%	0.116%	0.158%
2	2.655%	0.8445	1.547%	0.819%	0.290%	0.139%	0.151%
3	2.342%	1.1112	1.600%	0.692%	0.050%	0.119%	-0.069%
4	2.065%	0.8250	1.561%	0.362%	0.141%	0.065%	0.077%
5	1.923%	1.1935	1.679%	0.383%	-0.139%	-0.077%	-0.062%
6	2.469%	1.0248	2.112%	0.238%	0.120%	0.125%	-0.006%
7	2.262%	0.8573	1.982%	0.142%	0.138%	0.114%	0.024%
8	2.723%	1.1547	2.479%	0.121%	0.123%	0.139%	-0.016%
9	2.518%	1.1311	2.339%	0.058%	0.121%	0.128%	-0.007%
10	2.334%	0.9421	2.217%	-0.001%	0.118%	0.118%	0.000%

Some numerical insights III

Participating endowment, single recurrent premiums

Entry age: 50
Duration: 10 years
Annual premium amount: 1,000.00 euro
Pricing interest rate: $i = 2\%$
Pricing life table: period, population life table (mortality rates q)
Annual revaluation rate of the reserve: $r_t = \max\left\{\frac{0.95 \cdot i_t - i}{1+i}, 0\right\}$
Initial benefit amount (first premium): 1,216.06 euro

One trajectory for i_t and $q_{x+t}^{[\text{ref}]}$

t	i_t	$\frac{q_{x+t-1}^{[\text{ref}]}}{q_{x+t-1}}$	$i_t^{[\text{net}]}$	$\xi_t^{[\text{mut}]}$	$\xi_t^{[\text{load}]}$	$\xi_t^{[\text{load,fin}]}$	$\xi_t^{[\text{load,mort}]}$
1	2.256%	0.9196	2.105%	0.035%	0.116%	0.113%	0.003%
2	2.655%	0.8445	2.485%	0.031%	0.140%	0.134%	0.006%
3	2.342%	1.1112	2.190%	0.039%	0.114%	0.118%	-0.004%
4	2.065%	0.8250	1.967%	0.027%	0.070%	0.065%	0.006%
5	1.923%	1.1935	1.970%	0.036%	-0.083%	-0.077%	-0.006%
6	2.469%	1.0248	2.320%	0.025%	0.124%	0.125%	-0.001%
7	2.262%	0.8573	2.127%	0.018%	0.117%	0.114%	0.003%
8	2.723%	1.1547	2.571%	0.015%	0.136%	0.138%	-0.002%
9	2.518%	1.1311	2.384%	0.008%	0.127%	0.128%	-0.001%
10	2.334%	0.9421	2.217%	-0.001%	0.118%	0.118%	0.000%

In the following

- A risk pooling arrangement providing a post-retirement income ("annuity"), with (partial) financial and/or longevity guarantees
- Only mutuality returns and safety loadings (expenses and asset management fees are disregarded)
- Point of view of a survivor at time t, who holds information about:
  - The return on the investment, i_t
  - The individual account value ("policy reserve"), V_t
  - A mortality/longevity index

Equivalent periodic fees

The dynamics of the individual account value from the point of view of the survivor

$$V_t = \underbrace{V_{t-1}}_{\text{policy reserve}} \cdot (1 + \underbrace{i_t^{[\text{net}]}}_{\text{annual return, net of mutuality and loadings}}) - \underbrace{b_t}_{\text{Annuity benefit}}$$

...(Similar steps as for Insurance-based investment products)

Some numerical insights I

In this presentation:

- A deterministic assessment
- Tentative choice for the initial loading parameter
- One trajectory for i_t and $q^{[ref]}$
- Embedded fees in alternative arrangements

Some numerical insights II

Fixed-amount annuity

Entry age: 65	Initial benefit amount: 100.00 euro
Reserving interest rate: $i^{[\text{res}]} = 2\%$	Reserving life table: cohort, selected life table (mortality rates $q^{[\text{res}]}$)
Initial loading: $\lambda = 10\%$	Initial capital amount (single premium): 2,022.81 euro (annuity rate: 4.944%)
Minimum annual financial return: $i_{\min} = 2\%$	Financial participation rate: $\eta_t = 0\%$
	Longevity participation rate: $\gamma_t = 0\%$

t	i_t	$\frac{q^{[\text{bench}]}_{x+t-1}}{q^{[\text{ref}]}_{x+t-1}}$	b_t	$i^{[\text{net}]}_t$	$i^{[\text{mut}]}_t$	$\xi^{[\text{load}]}_t$	$\xi^{[\text{load,fin}]}_t$	$\xi^{[\text{load,mort}]}_t$	$\xi^{[\text{load,init}]}_t$
1	1.633%	1.057	100.00	2.062%	0.524%	0.095%	-0.367%	-0.030%	0.492%
5	2.450%	1.201	100.00	2.183%	0.615%	0.883%	0.450%	-0.123%	0.556%
10	1.768%	1.029	100.00	2.547%	1.182%	0.403%	-0.232%	-0.034%	0.669%
15	2.375%	1.379	100.00	3.319%	1.567%	0.623%	0.375%	-0.593%	0.841%
20	1.836%	1.390	100.00	5.367%	3.212%	-0.319%	-0.164%	-1.254%	1.099%
25	2.032%	1.325	100.00	9.764%	6.955%	-0.776%	0.032%	-2.263%	1.454%
30	2.435%	1.097	100.00	14.909%	13.483%	1.009%	0.435%	-1.305%	1.879%

Some numerical insights III

Participating annuity (financial participation)

Entry age: 65	Initial benefit amount: 100.00 euro
Reserving interest rate: $i^{[\text{res}]} = 2\%$	Reserving life table: cohort, selected life table (mortality rates $q^{[\text{res}]}$)
Initial loading: $\lambda = 5\%$	Initial capital amount (single premium): 1,930.87 euro (annuity rate: 5.179%)
Minimum annual financial return: $i_{\min} = 1\%$	Financial participation rate: $\eta_t = 95\%$
	Longevity participation rate: $\gamma_t = 0\%$

t	i_t	$\frac{q^{[\text{bench}]}_{x+t-1}}{q^{[\text{ref}]}_{x+t-1}}$	b_t	$i^{[\text{net}]}_t$	$i^{[\text{mut}]}_t$	$\xi^{[\text{load}]}_t$	$\xi^{[\text{load,fin}]}_t$	$\xi^{[\text{load,mort}]}_t$	$\xi^{[\text{load,init}]}_t$
1	1.633%	1.057	99.56	1.847%	0.523%	0.308%	0.082%	-0.030%	0.256%
5	2.450%	1.201	99.76	2.779%	0.619%	0.291%	0.123%	-0.124%	0.292%
10	1.768%	1.029	98.84	2.547%	1.182%	0.404%	0.088%	-0.034%	0.349%
15	2.375%	1.379	99.26	3.989%	1.577%	-0.037%	0.119%	-0.597%	0.442%
20	1.836%	1.390	98.68	5.647%	3.221%	-0.591%	0.092%	-1.257%	0.574%
25	2.032%	1.325	97.96	10.444%	6.998%	-1.414%	0.102%	-2.277%	0.761%
30	2.435%	1.097	98.05	16.293%	13.646%	-0.212%	0.122%	-1.321%	0.987%

Some numerical insights IV

Longevity-linked annuity

Entry age: 65	Initial benefit amount: 100.00 euro
Reserving interest rate: $i^{[\text{res}]} = 2\%$	Reserving and benchmark life table: cohort, selected life table (mortality rates $q^{[\text{res}]} = q^{[\text{bench}]}$)
Initial loading: $\lambda = 5\%$	Initial capital amount (single premium): 1,930.87 euro (annuity rate: 5.179%)
Minimum annual financial return: $i_{\text{min}} = 2\%$	Financial participation rate: $\eta_t = 0\%$
Bounds for the longevity revaluation rate: $r^{[\text{long}]}_{\text{min}} = -10\%$, $r^{[\text{long}]}_{\text{max}} = +10\%$	Longevity participation rate: $\gamma_t = 95\%$

t	i_t	$\frac{q^{[\text{bench}]}_{x+t-1}}{q^{[\text{ref}]}_{x+t-1}}$	b_t	$i^{[\text{net}]}_t$	$i^{[\text{mut}]}_t$	$\xi^{[\text{load}]}_t$	$\xi^{[\text{load,fin}]}_t$	$\xi^{[\text{load,mort}]}_t$	$\xi^{[\text{load,init}]}_t$
1	1.633%	1.057	99.97	2.269%	0.525%	-0.111%	-0.367%	-0.001%	0.257%
5	2.450%	1.201	99.75	2.331%	0.616%	0.735%	0.450%	-0.006%	0.291%
10	1.768%	1.029	99.11	2.837%	1.186%	0.117%	-0.232%	-0.002%	0.350%
15	2.375%	1.379	97.96	3.154%	1.564%	0.786%	0.375%	-0.027%	0.438%
20	1.836%	1.390	96.14	4.678%	3.191%	0.348%	-0.164%	-0.056%	0.569%
25	2.032%	1.325	91.14	8.209%	6.857%	0.680%	0.032%	-0.098%	0.746%
30	2.435%	1.097	87.04	14.519%	13.438%	1.354%	0.435%	-0.054%	0.972%

Some numerical insights V

Participating longevity-linked annuity
(financial participation and longevity linking)

Entry age: 65	Initial benefit amount: 100.00 euro
Reserving interest rate: $i^{[\text{res}]} = 2\%$	Reserving and benchmark life table: cohort, selected life table (mortality rates $q^{[\text{res}]} = q^{[\text{bench}]}$)
Initial loading: $\lambda = 2\%$	Initial capital amount (single premium): 1,875.70 euro (annuity rate: 5.331%)
Minimum annual financial return: $i_{\text{min}} = 1\%$	Financial participation rate: $\eta_t = 95\%$
Bounds for the longevity revaluation rate: $r^{[\text{long}]}_{\text{min}} = -10\%$, $r^{[\text{long}]}_{\text{max}} = +10\%$	Longevity participation rate: $\gamma_t = 95\%$

t	i_t	$\frac{q^{[\text{bench}]}_{x+t-1}}{q^{[\text{ref}]}_{x+t-1}}$	b_t	$i^{[\text{net}]}_t$	$i^{[\text{mut}]}_t$	$\xi^{[\text{load}]}_t$	$\xi^{[\text{load,fin}]}_t$	$\xi^{[\text{load,mort}]}_t$	$\xi^{[\text{load,init}]}_t$
1	1.633%	1.057	99.53	1.971%	0.523%	0.186%	0.082%	-0.001%	0.106%
5	2.450%	1.201	99.51	2.833%	0.619%	0.237%	0.123%	-0.006%	0.120%
10	1.768%	1.029	97.96	2.722%	1.184%	0.231%	0.088%	-0.002%	0.144%
15	2.375%	1.379	97.24	3.677%	1.572%	0.271%	0.119%	-0.029%	0.181%
20	1.836%	1.390	94.88	4.764%	3.194%	0.265%	0.092%	-0.060%	0.234%
25	2.032%	1.325	89.28	8.614%	6.882%	0.301%	0.102%	-0.108%	0.307%
30	2.435%	1.097	85.34	15.530%	13.556%	0.462%	0.122%	-0.061%	0.402%

Some remarks

Motivation

- Improve understanding of the benefit features, the size of their cost and get an insight into loadings
- Provide a description to individuals about the return and costs, in an "intuitive" way (and in line with indexes required for some financial contracts)
- Reinterpret actuarial fairness when individuals are only able to perform naive assessments
- Identify areas of improvement of the pricing process or policy design
- Describe the reserving process in an intuitive way against complex valuation rules (from local GAAP to IFRS17 and Solvency 2)
- Design products better meeting individuals' preferences

More investigation to follow . . .

Today's journey

References

This presentation is mainly based on:

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